Optimization of Organic Waste Blends for Efficient Anaerobic Digestion in Wastewater Treatment Systems Using a Circular Economy-Based Mathematical Model

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Abstract:

Organic waste generated during wastewater treatment, particularly sewage sludge, represents a significant resource with untapped material and energy potential. However, mono-digestion of sludge often results in suboptimal anaerobic digestion performance due to imbalanced substrate composition. This study presents the application of a mathematical model designed to determine optimal mixtures of sewage sludge and locally available organic substrates originating from agricultural activities and human-related processes in the vicinity of wastewater treatment plants. The developed model integrates quantitative and compositional characteristics of substrates, with particular emphasis on the C/N ratio, biodegradable organic content, and system material–energy requirements. By applying mass and energy balance constraints, the model identifies blending scenarios that maximize process stability, biogas production potential, and nutrient recovery while minimizing material losses.

In this paper, the focus is placed on the identification and analysis of optimal substrate mixtures, while the development of a fully integrated circular system and closed-loop implementation will be addressed in future research. Multiple management scenarios were evaluated to assess the degree of material inclusion, circularity indicators, and overall system efficiency. Results demonstrate that co-digestion strategies optimized through mathematical modeling significantly improve anaerobic digestion performance compared to sludge mono-digestion. The integration of locally generated organic residues enhances carbon balance, stabilizes the digestion process, and increases energy recovery potential.

The proposed approach represents a foundational step toward the transition from linear sludge management to integrated circular systems, providing a transferable and adaptable basis for further model development and system integration under circular economy principles in diverse territorial contexts.

Keywords:

Anaerobic digestion; Organic waste co-digestion; Circular economy; Mathematical modeling; C/N ratio optimization.

1. Introduction

The increasing amounts of organic waste and the need for its sustainable management represent a significant challenge for modern society. One important stream of organic waste is sludge generated during wastewater treatment processes. With the development of urban areas and the widespread implementation of wastewater treatment plants, the quantities of this sludge are continuously increasing. At the same time, due to its high organic matter content, sludge represents a potentially valuable resource that can be utilized for energy production.

One of the most applied methods for treating such waste is anaerobic digestion. In this process, organic matter is decomposed in the absence of oxygen, resulting in the production of biogas that can be used as a renewable energy source. However, when digestion is carried out using only wastewater sludge, reduced process efficiency is often observed. The main reason lies in the unfavorable composition of the substrate, particularly in terms of the carbon-to-nitrogen ratio, which can limit the activity of microorganisms responsible for organic matter degradation. This issue highlights a research gap in the development of systematic approaches that enable optimization of substrate composition in accordance with specific conditions and available resources.

For this reason, increasing attention is being given to co-digestion, that is, the combination of sludge with other organic substrates such as agricultural residues, biowaste, or by-products from the food industry. This approach improves substrate composition, ensures a more favorable C/N ratio, and creates more stable conditions for the anaerobic digestion process, which can lead to increased biogas production and more efficient utilization of organic matter. However, in practice, the selection and combination of appropriate substrates are often based on empirical approaches, without sufficiently developed tools for systematic and quantitative optimization.

In modern resource management approaches, the concept of circular economy is gaining increasing importance, emphasizing the efficient use of available resources and the reduction of waste through the reuse of materials and energy. In this context, integrating different sources of organic waste into a unified treatment system represents an important step toward more sustainable waste management. The significance of this approach is particularly reflected in the possibility of integrating locally available substrates into functional systems that simultaneously meet material and energy demands.

This paper presents the development of a mathematical model based on circular economy principles, which enables the optimization of blending ratios between organic substrates and wastewater treatment sludge in the anaerobic digestion process. The aim of this study is to develop and apply a model that identifies optimal substrate combinations to improve process performance and increase system circularity. In this paper, the focus is placed on the identification and formulation of optimal mixtures, while further research will be directed toward the development of a fully integrated closed-loop system. The model takes into account the quantities and composition of available substrates, with particular emphasis on the C/N ratio, as well as the material and energy balances of the system. Unlike conventional approaches, the developed model integrates multiple criteria, including circularity indicators, enabling a more comprehensive evaluation of different organic waste management scenarios.Based on these parameters, optimal substrate combinations are determined to contribute to more stable process performance, higher biogas production potential, and more efficient nutrient recovery.

2. Metodology

Within the research conducted during the development of the doctoral dissertation, as well as based on previously published scientific papers [1–4], a mathematical model for optimizing the blending ratios of different organic substrates was developed. The development of the model is based on the need to ensure an optimal composition of the substrate mixture used in the anaerobic digestion process, considering that the composition and characteristics of individual organic substrates significantly affect process stability and the efficiency of biogas production. In addition, the model development was guided by the need to establish a systematic and reproducible approach for substrate selection and combination under varying operational and environmental conditions.

The model was developed using a multi-criteria optimization approach, employing the global criterion of the Pareto optimal solution. This approach enables the simultaneous consideration of multiple parameters influencing the quality of the substrate mixture and the identification of optimal solutions that represent a compromise between defined criteria. As the main optimization criteria, the model incorporates the C/N ratio of the final mixture within the range of 20 to 30 and the moisture content of each individual substrate. These parameters play a crucial role in the anaerobic digestion process, as they directly influence microbial activity, process stability, and biogas production potential. Furthermore, their inclusion allows for a more realistic representation of process conditions, ensuring that the obtained solutions are applicable in practical wastewater treatment and waste management systems.

The mathematical model enables the analysis of various combinations of organic substrates and the determination of their optimal blending ratios in order to achieve the desired values of the selected criteria. In doing so, the characteristics of each individual substrate, as well as their contribution to the overall mixture composition, are taken into account. The application of the global Pareto optimality criterion allows for the identification of solutions in which no single system characteristic can be improved without compromising another, thereby defining the set of optimal substrate combinations. This provides decision-makers with a spectrum of feasible solutions, rather than a single fixed outcome, increasing flexibility in process design and management.

In this way, the developed mathematical model represents an efficient tool for optimizing substrate mixture composition and planning the co-digestion process of various organic substrates. Its application enables more rational utilization of available organic resources, improved anaerobic digestion performance, and more efficient organic waste management. In the context of this research, the model is primarily used for the identification and evaluation of optimal mixtures, while its further development toward integration into a comprehensive circular, closed-loop system will be the subject of future investigations.

3. Optimization Of Blending Ratios Of Organic Substrates With Wastewater Treatment Sludge

Within the optimization of blending ratios of organic substrates with wastewater treatment sludge, substrates that are realistically available within the territory of the Republic of Serbia were considered, originating from the immediate or wider vicinity of wastewater treatment plants (WWTPs). The selection of substrates was based on their availability, the quantities generated, and their potential for application in the anaerobic digestion process. Additionally, the selection criteria included their compatibility with sludge characteristics and their contribution to improving key process parameters, particularly the C/N ratio and moisture balance.

As potential organic substrates, agricultural residues, municipal waste, and animal manure were analyzed. Within agricultural residues, straw was examined as one of the most abundant lignocellulosic residues in agricultural production. From municipal activities, the organic fraction of municipal solid waste (OFMSW) was considered, while pig manure was included as a significant by-product of livestock production. In addition to these substrates, sludge generated during wastewater treatment was included in the model as the primary substrate whose co-digestion with other organic materials is being optimized. This selection ensures representation of the most relevant and abundantly available organic waste streams, enabling realistic modeling of substrate integration at the regional level.

Table 1. *Elemental composition and estimated quantities of selected organic substrates in the Republic of Serbia [5–10]*

Organic substrate	*C [%]*	*N [%]*	*P [%]*	*K [%]*	*Moisture [%]*	*Estimated quantity [t/year]*
Straw	42.00	0.45	2.65	14.84	10.80	1,458,208.20
OFMSW	47.90 38	3.10	0.28	0.78	70.00	526,918.08
Pig manure	39.14	3.92	3.69	0.74	82.50	4,668,800.00
WWTP sludge	30.00–45.00	3.00–6.00	1.50–3.00	0.20–0.50	75.00–85.00	32,932.38

For each of the considered organic substrates, the main physicochemical characteristics relevant to the anaerobic digestion process are presented. Table 1 shows the elemental composition of the substrates (C, N, P, and K), moisture content, and the estimated quantities of these materials within the Republic of Serbia. The estimation of quantities was based on literature data regarding crop yields [5, 7, 8, 9, 10], as well as statistical data from the Statistical Office of the Republic of Serbia [6]. These data provide a reliable basis for defining realistic input parameters and boundary conditions for the mathematical model.

Based on these data, input parameters for the mathematical model used to optimize substrate blending ratios were defined. Within the scope of this study, the model is specifically applied to evaluate and identify optimal mixture compositions, while further development will focus on integrating these results into a broader circular system with closed-loop material and energy flows.

4. Results and Discussion

The results obtained from the application of the developed mathematical model for optimizing the blending ratios of organic substrates with wastewater treatment sludge are presented in Table 2. The table shows the optimal mixing ratios of the considered substrates determined using the global Pareto optimality criterion, where the optimization criteria were the C/N ratio and the moisture content of the mixture. The application of this approach enabled the identification of a solution that simultaneously satisfies multiple process requirements, representing a balanced compromise between competing parameters.

Table 2. *Optimal mixing ratios of organic substrates with sewage sludge and characteristics of the mixture obtained using the mathematical model*

Organic substrate	*Share in mixture [%]*	*C/N ratio*	*Moisture [%]*
Straw	41.70	23.86	49.63
OFMSW	18.60		
Pig manure	29.80		
WWTP sludge	9.90		

Based on the defined input parameters for individual substrates, the model enabled the determination of substrate combinations that ensure the most favorable characteristics of the mixture for the anaerobic digestion process. In addition to the optimal mixing ratios of individual substrates, Table 2 also presents the resulting characteristics of the mixture, including the C/N ratio and moisture content, which represent key parameters for stable process performance. The obtained C/N ratio of 23.86 falls within the optimal range for anaerobic digestion (20–30), indicating favorable conditions for microbial activity and efficient organic matter degradation.

The results indicate that an appropriate combination of organic substrates with wastewater treatment sludge can achieve both a desirable C/N ratio and adequate moisture content of the mixture. The relatively high share of straw (41.70%) contributes to increasing the carbon content of the mixture, compensating for the nitrogen-rich components such as pig manure and sludge. At the same time, substrates with high moisture content, such as pig manure and OFMSW, play a key role in adjusting the overall moisture level of the mixture, ensuring suitable conditions for microbial processes.

A detailed analysis of the obtained mixture composition reveals the complementary roles of individual substrates. Straw, as a lignocellulosic material with high carbon content and low moisture, primarily contributes to balancing the C/N ratio, while pig manure, characterized by higher nitrogen content, enhances nutrient availability for microbial growth. OFMSW provides both biodegradable organic matter and moisture, supporting process kinetics, whereas sludge serves as the base substrate, ensuring continuity with wastewater treatment processes.

Furthermore, the results highlight the importance of integrating multiple substrates to overcome the limitations of mono-digestion. In the case of sludge-only digestion, suboptimal C/N ratios and excessive moisture can lead to process instability, reduced methane yields, and potential inhibition of microbial activity. The optimized mixture addresses these challenges by creating a balanced substrate composition that promotes stable digestion conditions.

From a process perspective, the achieved moisture content of 49.63% indicates that the mixture is well-suited for anaerobic digestion, avoiding both excessive dilution and insufficient water availability, which could negatively affect mass transfer and microbial activity. This balance is particularly important for maintaining optimal reactor performance and ensuring consistent biogas production.

In addition to improving process stability, the optimized mixture composition has significant implications for resource efficiency and circular economy implementation. By combining locally available organic residues, the model supports the utilization of diverse waste streams, reducing dependency on single substrates and minimizing material losses. This approach contributes to enhanced carbon utilization, improved nutrient recycling, and increased energy recovery potential in the form of biogas.

It is important to emphasize that, within the scope of this study, the presented results focus on the identification and evaluation of optimal substrate mixtures. While the obtained mixtures demonstrate clear advantages in terms of process performance, their integration into a fully closed-loop system, including nutrient recirculation and energy utilization pathways, remains a subject of future research.

Overall, the results confirm that the application of the developed mathematical model provides a robust and reliable tool for optimizing substrate mixtures in anaerobic digestion processes. The ability to systematically evaluate multiple substrate combinations and identify optimal solutions represents a significant advancement compared to conventional empirical approaches, offering a strong foundation for further development of integrated and circular waste management systems.

5. Conclusion

The application of the developed mathematical model in this study enabled the optimization of blending ratios of organic substrates with wastewater treatment sludge in order to achieve favorable conditions for the anaerobic digestion process. The model is based on the application of the global Pareto optimality criterion, with the C/N ratio and the moisture content of the substrate mixture used as the main optimization parameters. Such an approach allows for the simultaneous consideration of multiple process requirements, ensuring that the obtained solution represents a balanced and practically applicable compromise between key operational conditions.

Based on the characteristics of the considered organic substrates and their estimated quantities within the Republic of Serbia, optimal mixing ratios of straw, the organic fraction of municipal solid waste, pig manure, and wastewater treatment sludge were determined. The resulting optimal mixture, presented in Table 2, provides a C/N ratio of 23.86, which falls within the optimal range for stable anaerobic digestion. At the same time, the moisture content of the mixture is 49.63%, representing favorable conditions for further processing and adaptation within the digester. These results confirm that the developed model successfully captures the interactions between different substrate properties and translates them into operationally relevant mixture compositions.

The research results indicate that an appropriate combination of different organic substrates can achieve a balanced composition of the substrate mixture, thereby improving conditions for the microbiological degradation of organic matter and increasing the potential for biogas production. In particular, the integration of carbon-rich and nitrogen-rich substrates enables effective regulation of the C/N ratio, while the combination of materials with varying moisture contents ensures suitable physical conditions for microbial activity and mass transfer processes. The application of the developed mathematical model enables efficient analysis of different substrate blending scenarios and represents a useful tool for planning co-digestion processes in organic waste management systems. Moreover, the model provides a structured framework for decision-making, reducing reliance on empirical approaches and enabling more precise and reproducible process design.

In this way, more efficient utilization of available organic resources is achieved, along with improved sustainable waste management in accordance with circular economy principles. It is important to emphasize that, within the scope of this study, the model is primarily applied for the identification and evaluation of optimal substrate mixtures, while future research will focus on the integration of these results into a comprehensive closed-loop system, including nutrient recovery and energy valorization pathways.

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