# Two-Stage Distributed Optimization for Peer-to-Peer Energy Trading Combining Mixed-Integer Linear Programming and Nonlinear Programming

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**Abstract:**

Peer-to-peer (P2P) energy trading contributes to the enhancement of the self-consumption capability of PV output among prosumers and consumers. Optimal planning for P2P energy trading networks can typically be formulated in a mixed-integer nonlinear constrained programming problem, which is challenging to solve even using a commercial optimization solver. The present study develops a two-stage distributed optimization method for P2P energy trading networks. In this method, the original problem is decomposed into a first-stage coordinated operational planning problem without considering the P2P energy trading charge and a second-stage benefit allocation planning problem based on asymmetric Nash bargaining and the bargaining power calculated by the contribution to the P2P energy trading. The first-stage and second-stage problems result in a mixed-integer linear programming and convex nonlinear programming problems, respectively. To perform the two-stage optimization in a distributed manner from the perspective of information security and problem scalability, consensus ADMM (alternative direction method of multipliers) for multiple subproblems and the Release and Fix method for subproblems with integer variables are uniquely combined. The developed method is applied to the P2P energy trading network consisting of three prosumers with PV and battery, and one consumer. The results show that the developed method can provide fair allocation of the benefit by P2P energy trading. Moreover, a distributively optimal solution is close to the centrally optimal solution.

**Keywords:**

Peer-to-peer energy trading, Distributed optimization, Cooperative game theory, Mixed-integer linear programming, Nonlinear programming, Consensus ADMM.

# 1. Introduction

The introduction of renewable energy-based power generation is progressing; photovoltaic (PV) generation is its major one. However, the management of the PV output variations is a crucial issue to enhance the grid connection capacity of PV generation. Thus, the self-consumption of the PV output should be enhanced. A promising technology is the coordinated operation of multiple PV generation units on the demand side. A typical implementation is peer-to-peer (P2P) energy trading, in which multiple electricity trades are simultaneously performed in a peer-to-peer fashion among prosumers with PV generation and battery, and consumers [1]. Moreover, the energy cost of prosumers and consumers, who participate in the P2P energy trading, can be reduced by properly determining the energy trading price depending on the variations in the weather, power retailer price, and energy demand. Fair benefit allocation among prosumers and consumers is indispensable because their equipment capacity and configuration are different. This decision-making can be expressed as an operational planning problem with a given input scenario. The mathematical optimization problem typically results in a mixed-integer nonlinear constrained programming problem because the energy trading amount and price should be optimized simultaneously, and nonlinear calculation to evaluate the P2P participating contribution must be considered. Solving these problems with a commercial optimization solver is still challenging in large-scale problems with many integer variables and nonlinear terms. Moreover, these problems should be solved in a distributed manner from the perspective of information security of prosumers and consumers and the scalability of the optimization problems.

Previous studies, which tackle the above-mentioned optimization problems of P2P energy trading networks, focus on an algorithm-based benefit allocation [2] and a game theory-based benefit allocation [3–7]. The algorithm-based framework [2] cannot fairly allocate the benefit obtained by performing the P2P energy trading. For game theory-based studies, Zhang et al. [3] optimized benefit allocation using a Shapley value, which can achieve fair benefit allocation based on cost reduction contribution. However, the cost reduction must be evaluated in all combinations of the P2P energy trading. This results in a high computational cost in large-scale P2P energy trading networks. Wei et al. [4] optimized the energy trading planning based on Stackelberg competition. In this non-cooperative game theory, leaders make a trading price decision and followers make a trading amount decision based on the decided price; thus, leaders have an advantage over followers. Duan et al. [5] explored a Pareto front of benefit allocation planning of a P2P energy trading network using Nash bargaining, which is a cooperative game theory to achieve fair benefit allocation. To efficiently solve a non-convex nonlinear programming problem, they developed a two-stage optimization method, in which the original problem is decomposed into a coordinated operational planning problem and a benefit allocation planning problem. However, Nash bargaining cannot conduct a fair benefit allocation based on the cost reduction contribution of prosumers and consumers. Thus, Cai et al. [6] and Lv et al. [7] focused on asymmetric Nash bargaining, in which the cost reduction contribution of each prosumer and consumer can be evaluated as a bargaining power, in the second-stage benefit allocation planning problem.

Moreover, Duan et al. [5], Cai et al. [6], and Lv et al. [7] employed the alternating direction method of multipliers (ADMM) for distributed optimization of the P2P energy trading. The ADMM used by Duan et al. [6] and Cai et al. [7] was a conventional type, in which the original problem is decomposed into two convex subproblems. Thus, the convergence of the ADMM iterative calculation for multiple non-convex subproblems, including integer variables, is not guaranteed. Lv et al. [7] applied consensus ADMM, which can converge iterative calculation even in multiple convex subproblems. Furthermore, Cai et al. [6] combined the Release and Fix method [8] to explore a near-optimal solution of subproblems including integer variables through ADMM iterative calculation.

The present study develops a two-stage distributed optimization method for P2P energy trading networks to efficiently determine the energy trading amount and the energy trading price. The developed method is based on a two-stage optimization [5–7], in which the original problem is decomposed into a first-stage coordinated operational planning problem without considering the P2P energy trading charge and a second-stage benefit allocation planning problem based on the asymmetric Nash bargaining and the bargaining power calculated by the contribution to the P2P energy trading. The first-stage and second-stage problems result in a mixed-integer linear programming (MILP) and convex nonlinear programming (NLP) problems, respectively. To perform the two-stage optimization in a distributed manner from the perspective of information security and problem scalability, the consensus ADMM for multiple subproblems and the Release and Fix method for non-convex subproblems with integer variables are combined. To the best of our knowledge, no previous study on distributed optimization of P2P energy trading planning, which combines consensus ADMM with Release and Fix method, was found. The developed method is applied to the P2P energy trading network consisting of three prosumers with PV and battery, and one consumer. The results show that the developed method can provide fair allocation of the benefit by performing P2P energy trading. Moreover, a distributively optimal solution is close to the centrally optimal solution.

# 2. Peer-to-peer (P2P) energy trading network

An illustrative configuration of a P2P energy trading network is shown in Fig. 1. The P2P energy trading network is organized among prosumers and consumers. A consumer installs a heat pump water heater and a hot water storage tank. In prosumers, a PV panel and a battery unit can also be installed. The P2P energy trading indicates power exporting and importing between two prosumers or a prosumer and a consumer, which is conducted by using the PV surplus power and the battery discharging power. Multiple electricity trades are simultaneously performed in a P2P fashion. The power imported from other prosumers is used to supply to the electric power demand, operate the heat pump water heater, and charge the battery unit. The PV surplus power can also be sold to a power retailer. If the power demand is not met by the PV power, the battery

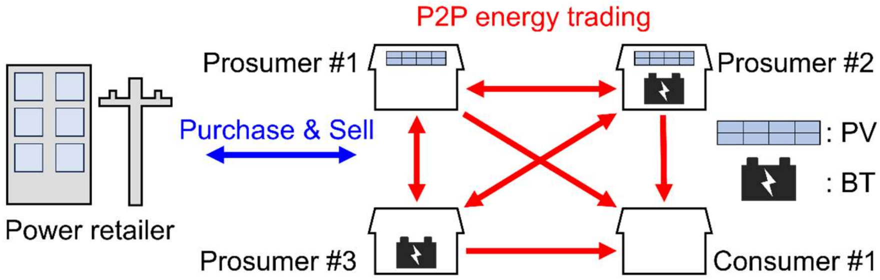



**Figure. 1.** Conceptual diagram of P2P energy trading network.

discharging power, and the importing power from other prosumers, the power shortfall can be individually purchased from a power retailer.

# 3. Two-stage optimization of operational planning of P2P energy trading network

## 3.1. Overview

In P2P energy trading networks, the energy trading amount and the energy trading price should be optimized simultaneously so as to maximize the benefit of the prosumers participating in the P2P energy trading. Moreover, the benefit should be fairly allocated on the basis of their participating contribution. This optimization problem results in a mixed-integer nonlinear constrained programming problem because the product of two decision variables, i.e., the energy trading amount and the energy trading price, and nonlinear calculation to evaluate participating contribution must be considered. As the number of participants in the P2P energy trading and the sampling times in the planning horizon increase, it would be challenging to solve this problem even using a commercial optimization solver.

The present study first develops a two-stage optimization method for operational planning of P2P energy trading networks, as shown in Fig. 2. The original mixed-integer nonlinear constrained programming problem is decomposed into a first-stage MILP-based coordinated operational planning problem and a second-stage NLP-based benefit allocation planning problem. In the first stage, the energy trading planning is optimized so as to minimize the total energy cost of the P2P energy trading network with no consideration of the P2P energy trading charge. The PV output, energy demand of the prosumers and consumers, and the time-variant purchasing and selling prices with a power retailer are given as the input conditions. This coordinated operational planning optimization problem is formulated as a MILP problem. Then, the contribution to the P2P energy trading of all the prosumers and consumers is evaluated by the result of the coordinated operational planning. In the second stage, the benefit obtained by performing the P2P energy trading is fairly allocated by optimizing the energy trading price using asymmetric Nash bargaining, which considers the contribution to the P2P energy trading of the prosumers and consumers as bargaining power. The energy trading planning, the bargaining power, and the benefit, which were calculated in the first stage, are given as the input condition. This benefit allocation planning problem is formulated as a convex NLP problem. By employing this two-stage optimization method, the energy trading amount and price can be efficiently optimized.

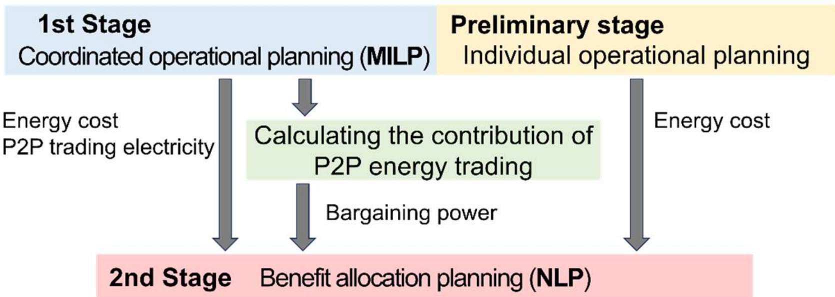



**Figure. 2.** Two-stage optimization method for operational planning of P2P energy trading network.

## 3.2. First-stage coordinated operational planning problem

In the first stage, the coordinated operational planning of the P2P energy trading network, i.e., the energy trading planning as well as the operational planning of all the prosumers and consumers at each sampling time in the planning horizon, is optimized by providing PV output, energy demand, and time-variant purchasing and selling prices of electricity with a power retailer. The objective function to be minimized is the total energy cost of the P2P energy trading network, which is the sum of the purchasing electricity cost and the selling electricity benefit of all the prosumers and consumers. The P2P energy trading charge among the prosumers and consumers is not considered in the first stage. Let us focus on a P2P energy trading network consisting of $N$ prosumers. The following formulated problem results in an MILP problem.

### 3.2.1. Decision variables

The decision variables are composed of binary and continuous variables. The binary variables express the operational states of energy supply units, power charging/discharging states of the storage units, purchasing/selling states with a power retailer, and power exporting/importing states among the prosumers at each sampling time. The continuous variables express input and output energy flow rates of energy supply units, stored energy of storage units, purchasing and selling power with a power retailer, and exporting and importing power among the prosumers at each sampling time.

### 3.2.2. Constraints

Performance characteristics of energy supply and storage units, energy balances, P2P energy trading constraints, and the calculation formula of energy cost are formulated in linear forms using decision variables.

In the P2P energy trading, power exporting and importing between two prosumers is established. The trading electric power between the *m*th and *n*th prosumers is defined as $e_{n,m,k}^{\mathrm{P2P}}$. A positive value of $e_{n,m,k}^{\mathrm{P2P}}$ denotes exporting power from the *n*th prosumer to the *m*th prosumer. Thus, the P2P energy trading constraint between *m*th and *n*th prosumers at *k*th sampling time is expressed by the following equation:

$$\left.\begin{array}{l} e_{n,m,k}^{\mathrm{P2P}} + e_{m,n,k}^{\mathrm{P2P}} = 0 \\ \underline{E}\delta_{n,m,k}^{\mathrm{I}} \le e_{n,m,k}^{\mathrm{P2P}} \le \bar{E}\delta_{n,m,k}^{\mathrm{E}} \\ \delta_{n,m,k}^{\mathrm{E}}, \delta_{n,m,k}^{\mathrm{I}} \in \{0,1\} \end{array}\right\}, \quad \forall n \in N, \forall m \in N, \forall k \in K \tag{1}$$

where $\delta_{n,m,k}^{\mathrm{E}}$ and $\delta_{n,m,k}^{\mathrm{I}}$ are the binary variables denoting the power exporting and importing states in the corresponding P2P energy trading, respectively. The P2P energy trading constraint must be further integrated with the power exporting/importing states at each prosumer. By defining $\delta_{n,k}^{\mathrm{E}}$ and $\delta_{n,k}^{\mathrm{I}}$ as the binary variables denoting the power exporting and importing states at the *n*th prosumer, respectively, the power exporting/importing constraint at each prosumer can be expressed as follows:

$$\left.\begin{array}{l} e_{n,k}^{\mathrm{E}} - e_{n,k}^{\mathrm{I}} = \sum_{m \in N} e_{n,m,k}^{\mathrm{P2P}} \\ \delta_{n,k}^{\mathrm{E}} + \delta_{n,k}^{\mathrm{I}} \le 1 \\ \delta_{n,m,k}^{\mathrm{E}} \le \delta_{n,k}^{\mathrm{E}}, \quad \forall m \in N \\ \delta_{n,m,k}^{\mathrm{I}} \le \delta_{n,k}^{\mathrm{I}}, \quad \forall m \in N \\ \delta_{n,k}^{\mathrm{E}}, \delta_{n,k}^{\mathrm{I}} \in \{0,1\} \end{array}\right\}, \quad \forall n \in N, \forall k \in K \tag{2}$$

where $e_{n,k}^{\mathrm{E}}$ and $e_{n,k}^{\mathrm{I}}$ are the exporting and importing power at the *n*th prosumer, respectively.

By including $e_{n,k}^{\mathrm{E}}$ and $e_{n,k}^{\mathrm{I}}$, the power balance at each prosumer is formulated as follows:

$$E_{n,k}^{\mathrm{PV}} + e_{n,k}^{\mathrm{BT,O}} + e_{n,k}^{\mathrm{P}} + e_{n,k}^{\mathrm{I}} = E_{n,k}^{\mathrm{D}} + e_{n,k}^{\mathrm{HP}} + e_{n,k}^{\mathrm{BT,I}} + e_{n,k}^{\mathrm{S}} + e_{n,k}^{\mathrm{E}}, \quad \forall n \in N, \forall k \in K \tag{3}$$

where $E_{n,k}^{\mathrm{PV}}$ and $E_{n,k}^{\mathrm{D}}$ are the parameters expressing the PV power and power demand, respectively; $e_{n,k}^{\mathrm{BT,O}}$ and $e_{n,k}^{\mathrm{BT,I}}$ are the battery discharging and charging power, respectively; $e_{n,k}^{\mathrm{HP}}$ is the input power to the heat pump water heater; and $e_{n,k}^{\mathrm{P}}$ and $e_{n,k}^{\mathrm{S}}$ are the purchased and sold power with the power retailer, respectively.

The energy cost of each prosumer $c_n^{\mathrm{CO}}$ in the P2P energy trading network is calculated using the purchased and sold power with the power retailer $e_{n,k}^{\mathrm{P}}$ and $e_{n,k}^{\mathrm{S}}$, respectively, and the parameters expressing time-variant purchasing and selling prices with the power retailer $\Phi_{n,k}^{\mathrm{P}}$ and $\Phi_{n,k}^{\mathrm{S}}$, respectively, as follows:

$$c_n^{\mathrm{CO}} = \sum_{k \in K} \left(\Phi_{n,k}^{\mathrm{P}} e_{n,k}^{\mathrm{P}} - \Phi_{n,k}^{\mathrm{S}} e_{n,k}^{\mathrm{S}}\right)\Delta T, \quad \forall n \in N \tag{4}$$

For the details of the constraint formulation for performance characteristics of energy supply and storage units, refer to our previous publication [9]

### 3.2.3. Objective function

The objective function to be minimized is the total energy cost of the P2P energy trading network. This is the sum of the energy cost of each prosumer, as expressed by the following equation:

$$J_1 = \sum_{n \in N} c_n^{\mathrm{CO}} \tag{5}$$

It should be noted that the P2P energy trading charge among prosumers is not considered in the first stage because the benefit based on the P2P energy trading is fairly allocated in the second stage.

## 3.3. Evaluation of P2P energy trading contribution

To fairly allocate the benefit among the prosumers, the contribution to the P2P energy trading of each prosumer is evaluated from the results of the first-stage coordinated operational planning. The focused contributions are threefold: PV output utilization, engagement in the P2P energy trading, and battery utilization.

The first contribution is the PV output utilization ratio $R_n^{\mathrm{PV}}$, which is defined as the ratio of the electricity used for the P2P energy trading to the total amount of the PV generation, as follows:

$$R_n^{\mathrm{PV}} = \begin{cases} \sum_{k\in K} \frac{E_{n,k}^{\mathrm{PV,P2P}} \Delta T}{E_{n,k}^{\mathrm{PV}} \Delta T} & (E_n^{\mathrm{PV}} > 0) \\ 0 & \end{cases}, \quad \forall n \in N \tag{6}$$

where $E_{n,k}^{\mathrm{PV,P2P}}$ is the PV power used for the P2P energy trading, which was calculated in the first stage.

The second contribution is the engagement ratio in the P2P energy trading $R_n^{\mathrm{P2P}}$, which is defined as the ratio of the amount of the actual traded electricity to the total amount of P2P energy trading as follows:

$$R_n^{\mathrm{P2P}} = \begin{cases} \sum_{k\in K} \frac{\left(E_{n,k}^{\mathrm{E}} + E_{n,k}^{\mathrm{I}}\right)\Delta T}{\sum_{n\in N}\left(E_{n,k}^{\mathrm{E}} + E_{n,k}^{\mathrm{I}}\right)\Delta T} & \left(E_{n,k}^{\mathrm{E}}, E_{n,k}^{\mathrm{I}} > 0\right) \\ 0 & \end{cases}, \quad \forall n \in N \tag{7}$$

where $E_{n,k}^{\mathrm{E}}$ and $E_{n,k}^{\mathrm{I}}$ are the exporting and importing powers, respectively, which were calculated in the first stage.

The third contribution is the battery utilization ratio $R_n^{\mathrm{BT}}$, which is defined as the ratio of the amount of the battery charging and discharging power used for the P2P energy trading to their total amount, as follows:

$$R_n^{\mathrm{BT}} = \begin{cases} \sum_{k\in K} \frac{\left(E_{n,k}^{\mathrm{BT,O,P2P}} + E_{n,k}^{\mathrm{BT,I,P2P}}\right)\Delta T}{\left(E_{n,k}^{\mathrm{BT,O}} + E_{n,k}^{\mathrm{BT,I}}\right)\Delta T} & \left(E_{n,k}^{\mathrm{BT,O}}, E_{n,k}^{\mathrm{BT,I}} \geq 0\right) \\ 0 & \end{cases}, \quad \forall n \in N \tag{8}$$

where $E_{n,k}^{\mathrm{BT,O}}$ and $E_{n,k}^{\mathrm{BT,I}}$ are the battery discharging and charging power, respectively; and $E_{n,k}^{\mathrm{BT,O,P2P}}$ and $E_{n,k}^{\mathrm{BT,I,P2P}}$ are the battery discharging and charging power used for the P2P energy trading, respectively. $E_{n,k}^{\mathrm{BT,O}}$, $E_{n,k}^{\mathrm{BT,I}}$, $E_{n,k}^{\mathrm{BT,O,P2P}}$, and $E_{n,k}^{\mathrm{BT,I,P2P}}$ were calculated in the first stage.

The comprehensive contribution to the P2P energy trading of each prosumer is evaluated by the following weighted sum of the three contributions:

$$\left.\begin{array}{l} R_n = \varphi_{\mathrm{PV}} R_n^{\mathrm{PV}} + \varphi_{\mathrm{P2P}} R_n^{\mathrm{P2P}} + \varphi_{\mathrm{BT}} R_n^{\mathrm{BT}}, \quad \forall n \in N \\ \varphi_{\mathrm{PV}} + \varphi_{\mathrm{P2P}} + \varphi_{\mathrm{BT}} = 1 \end{array}\right\} \tag{9}$$

where $\varphi_{\mathrm{PV}}$, $\varphi_{\mathrm{P2P}}$, and $\varphi_{\mathrm{BT}}$ are the weighting parameters.

The bargaining power of each prosumer to derive the asymmetric Nash bargaining solution in the second stage, $B_n$, is calculated by normalizing using the total sum of all the prosumers as follows:

$$B_n = \frac{R_n}{\sum_{n\in N} R_n}, \quad \forall n \in N \tag{10}$$

## 3.4. Second-stage benefit allocation planning problem

In the second stage, the benefit obtained by performing the P2P energy trading in the first stage is fairly allocated by optimizing the energy trading price using asymmetric Nash bargaining [10]. Unlike symmetric Nash bargaining, assuming a symmetric bargaining power, asymmetric Nash bargaining can incorporate bargaining powers depending on the P2P energy trading contribution of each prosumer; this results in a fair benefit allocation to prosumers. The input condition in the second-stage benefit allocation planning problem is the energy trading planning and the benefit by performing the P2P energy trading, which were calculated in the first stage (Section 3.2), and the bargaining power of each prosumer, which was evaluated in Subsection 3.3. Moreover, the individual operational planning of each prosumer without P2P energy trading is preliminarily conducted. This preliminary-stage operational planning problem results in a MILP problem and is solved separately. The energy cost of the individual operation of prosumers is regarded as a non-participation benefit and is also given as the input condition. The second-stage optimization determines the time-variant energy trading price of each P2P energy trading so as to maximize an asymmetric Nash product expressing the product of the benefit of each prosumer weighted using bargaining powers. This benefit allocation planning problem results in a convex NLP problem.

### 3.4.1. Decision variables

The decision variables in the second stage are the time-variant energy trading price of each P2P energy trading $\psi_{n,m,k}^{\mathrm{P2P}}$, which are expressed by continuous variables.

### 3.4.2. Constraints

The constraints consist of the calculation of the P2P energy trading charge among the prosumers and the reduction in the energy cost of each prosumer from that in the individual operation.

The P2P energy trading charge of each prosumer $c_n^{\mathrm{P2P}}$ is calculated by using $\psi_{n,m,k}^{\mathrm{P2P}}$ and the energy trading planning result $E_{n,m,k}^{\mathrm{P2P}}$, which was calculated in the first stage, as follows:

$$c_n^{\mathrm{P2P}} = \sum_{m\in N}\sum_{k\in K}\left(-\psi_{n,m,k}^{\mathrm{P2P}} E_{n,m,k}^{\mathrm{P2P}}\right)\Delta T\ , \quad \forall n \in N \tag{11}$$

Positive and negative values of $c_n^{\mathrm{P2P}}$ denote expenses and income, respectively. The time-variant energy trading price must be coincident between the trading pair of the prosumers at each sampling time. This can be expressed by the following equation:

$$\psi_{n,m,k}^{\mathrm{P2P}} = \psi_{m,n,k}^{\mathrm{P2P}}\ , \quad \forall n \in N, \forall m \in N, \forall k \in K \tag{12}$$

Moreover, the reduction in the energy cost of each prosumer from that in the individual operation is guaranteed by the following constraint:

$$C_n^{\mathrm{CO}} + c_n^{\mathrm{P2P}} \le C_n^{\mathrm{IND}}\ , \quad \forall n \in N \tag{13}$$

where $C_n^{\mathrm{CO}}$ and $C_n^{\mathrm{IND}}$ are the parameters expressing the energy cost of each prosumer in the P2P energy trading network and the individual operation, respectively, which were calculated in the first and preliminary stages, respectively.

### 3.4.3. Objective function

The objective function is the asymmetric Nash product, which is the product of the benefit of each prosumer weighted using bargaining powers $B_n$.

$$J_2 = \prod_{n\in N}\left(C_n^{\mathrm{IND}} - C_n^{\mathrm{CO}} - c_n^{\mathrm{P2P}}\right)^{B_n} \tag{14}$$

# 4. Distributed optimization using consensus ADMM and Release & Fix method

## 4.1. Overview

The two-stage optimization of operational planning of P2P energy trading networks, developed in Section 3, should be performed in a distributed manner from the perspective of information security of prosumers and scalability of the optimization problems. Thus, the present study develops a distributed optimization method using consensus ADMM and the Release & Fix method, of which the framework is shown in Fig. 3.

When applying conventional ADMM to distributed optimization consisting of more than two subproblems, the convergence of ADMM iterative calculation is guaranteed only under the condition that the coefficient matrix of the constraints is diagonal [11]. Furthermore, its convergence is not always guaranteed in non-convex programming problems, such as MILP problems including integer variables [12]. Thus, the present study combines consensus ADMM, which can be applied to distributed optimization with multiple subproblems, and the Release and Fix method [8], which is a heuristic to find a near-optimal solution of MILP problems through ADMM iterative calculation.

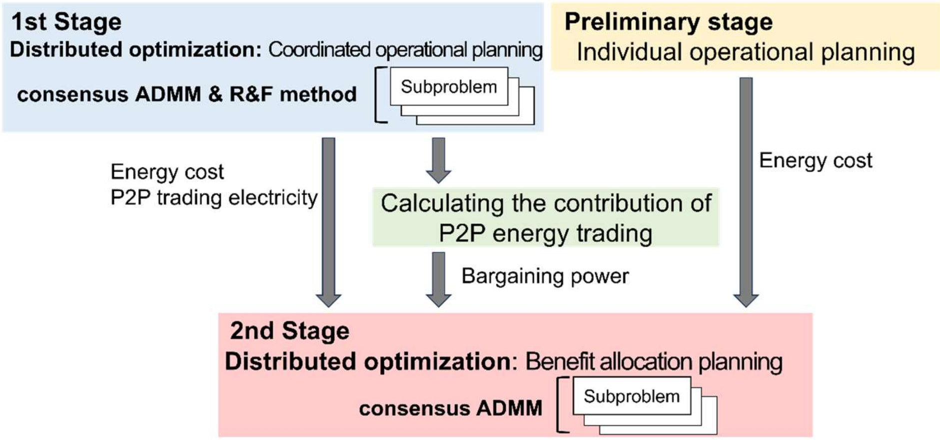



**Figure. 3.** Two-stage distributed optimization method for operational planning of P2P energy trading network.

## 4.2. Consensus ADMM

Consensus ADMM is an extension of conventional ADMM so as to converge the solution in the distributed optimization with more than two subproblems [13]. To apply the distributed optimization of the first-stage problem in Subsection 3.2, the original problem is decomposed into *N* subproblems; each subproblem is considered for each prosumer. It should be noted that the number of P2P energy trading path among the prosumer is ${}_N C_2$. In consensus ADMM, the set of P2P trading power of the *n*th prosumer to the *i*th trading path in the planning horizon, $\boldsymbol{e}_{n,i}$, is regarded as local variables in each subproblem. Moreover, the target value of $\boldsymbol{e}_{n,i}$ is defined as global variables $\boldsymbol{z}_{n,i}$, which are shared and exchanged among subproblems. Unlike conventional ADMM, a consensus constraint to match the value of $\boldsymbol{e}_{n,i}$ with the value of $\boldsymbol{z}_{n,i}$ is introduced in consensus ADMM. By incorporating this consensus constraint with an augmented Lagrangian and alternately updating local variables, including $\boldsymbol{e}_{n,i}$, and global variables $\boldsymbol{z}_{n,i}$, distributed optimization can be performed in each subproblem (prosumer).

The first-stage coordinated operational planning problem in Subsection 3.2 can be reformulated in a distributed manner as follows:

$$\left.\begin{aligned} &\min_{\boldsymbol{x}} \sum_{n\in N} f_n(\boldsymbol{x}_n) \\ &\text{s.t. } \boldsymbol{x}_n \in \mathcal{X}_n, \quad \forall n \in N \\ &\qquad \sum_{n\in S_i} \boldsymbol{e}_{n,i} = 0, \quad \forall i \in I \end{aligned}\right\} \tag{15}$$

where $\boldsymbol{x}$ is the vector expressing all the decision variables; $\mathcal{X}$ is the set of the decision variables satisfying all the constraints; $f$ is the calculation formula of the objective function; $\boldsymbol{e}$ is the vector expressing the P2P trading power of all the prosumers and included in $\boldsymbol{x}$; $I$ and $i$ are the set of trading path ( $|I|$ = ${}_N C_2$) and its index, respectively; and $S_i$ is the set of prosumers involving $i$th trading path. The second constraint is the coupling constraint among all the prosumers, which denotes the aggregation of the P2P energy trading constraint shown in the first equation of Eq. (1).

By redundantly introducing the global variables regarding the P2P trading power $\boldsymbol{z}_{n,i}$, the original problem can be reformulated as follows:

$$\left.\begin{aligned} &\min_{\boldsymbol{x},\boldsymbol{z}} \sum_{n\in N} f_n(\boldsymbol{x}_n) \\ &\text{s.t. } \boldsymbol{x}_n \in \mathcal{X}_n, \quad \forall n \in N \\ &\qquad \boldsymbol{e}_{n,i} = \boldsymbol{z}_{n,i}, \quad \forall n \in N, \forall i \in I \\ &\qquad \sum_{n\in S_i} \boldsymbol{z}_{n,i} = 0, \quad \forall i \in I \end{aligned}\right\} \tag{16}$$

The second and third constraints in Eq. (16) are the consensus and coupling constraints, respectively.

In this problem, the augmented Lagrangian incorporating the consensus constraint can be expressed by the following equation:

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{\alpha})=\sum_{n\in N}\left\{f_n(\boldsymbol{x}_n)+\sum_{i\in I}\boldsymbol{\alpha}_{n,i}^{\mathrm{T}}\left(\boldsymbol{e}_{n,i}-\boldsymbol{z}_{n,i}\right)+\sum_{i\in I}\frac{\rho}{2}\left\|\boldsymbol{e}_{n,i}-\boldsymbol{z}_{n,i}\right\|_2^2\right\} \tag{17}$$

where $\boldsymbol{\alpha}$ is the vector expressing the Lagrange multiplier [JPY/kWh]; and $\rho$ is the penalty coefficient [JPY/(kWh)$^2$]. This augmented Lagrangian is apparently decomposable into each subproblem (prosumer).

Based on Reference [12], the algorithm to sequentially update the local variables $\boldsymbol{x}_n$, global variables $\boldsymbol{z}_{n,i}$, and Lagrange multiplier $\boldsymbol{\alpha}_{n,i}$ can be expressed as follows:

$$\boldsymbol{x}_n^{t+1}=\underset{\boldsymbol{x}_n\in\mathcal{X}_n}{\arg\min}\left\{f_n(\boldsymbol{x}_n)+\sum_{i\in I}\left(\boldsymbol{\alpha}_{n,i}^t\right)^{\mathrm{T}}\boldsymbol{e}_{n,i}+\sum_{i\in I}\frac{\rho}{2}\left\|\boldsymbol{e}_{n,i}-\boldsymbol{z}_{n,i}^t\right\|_2^2\right\},\quad \forall n\in N \tag{18}$$

$$\boldsymbol{z}_i^{t+1}=\underset{\boldsymbol{z}_i\in Z_i}{\arg\min}\sum_{n\in S_i}\left\{-\left(\boldsymbol{\alpha}_{n,i}^t\right)^{\mathrm{T}}\boldsymbol{z}_{n,i}+\frac{\rho}{2}\left\|\boldsymbol{e}_{n,i}^{t+1}-\boldsymbol{z}_{n,i}\right\|_2^2\right\},\quad \forall i\in I \tag{19}$$

$$\boldsymbol{\alpha}_{n,i}^{t+1}=\boldsymbol{\alpha}_{n,i}^t+\rho\left(\boldsymbol{e}_{n,i}^{t+1}-\boldsymbol{z}_{n,i}^{t+1}\right),\quad \forall n\in N,\forall i\in I \tag{20}$$

where $t$ is the iteration number, and $Z$ is the set of the global variables satisfying the coupling constraint of Eq. (16). Moreover, the updating problem of $\boldsymbol{z}_i$ in Eq. (19) can be expressed by solving with the Lagrange multiplier method as follows:

$$\boldsymbol{z}_{n,i}^{t+1}=\boldsymbol{e}_{n,i}^{t+1}-\frac{1}{N}\sum_{n\in S_i}\boldsymbol{e}_{n,i}^{t+1}=\boldsymbol{e}_{n,i}^{t+1}-\bar{\boldsymbol{e}}_i^{t+1},\quad \forall n\in N,\forall i\in I \tag{21}$$

where $\bar{\boldsymbol{e}}$ is the vector expressing the averaging value of P2P trading power.

By providing $\boldsymbol{z}_{n,i}^t$ and $\boldsymbol{\alpha}_{n,i}^t$ as parameters, $\boldsymbol{x}_n^{t+1}$ is updated with Eq. (18) because $\boldsymbol{e}_{n,i}$ is included in $\boldsymbol{x}_n$. Then, $\boldsymbol{z}_{n,i}^{t+1}$ is updated by using Eq. (21) and $\boldsymbol{e}_{n,i}^{t+1}$, of which the values are exchanged among the prosumers. Finally, $\boldsymbol{\alpha}_{n,i}^{t+1}$ is updated by using Eq. (20), $\boldsymbol{e}_{n,i}^{t+1}$, and $\boldsymbol{z}_{n,i}^{t+1}$, and then $\boldsymbol{x}_n^{t+2}$ is sequentially updated. Obviously, updating $\boldsymbol{x}_n$, $\boldsymbol{z}_{n,i}$, and $\boldsymbol{\alpha}_{n,i}$ can be separately conducted in each subproblem (prosumer).

The primal and dual residuals for the convergence test are expressed by the following equations:

$$\boldsymbol{r}^t=\sum_{i\in I}\sum_{n\in S_i}\boldsymbol{e}_{n,i}^t \tag{22}$$

$$\boldsymbol{s}^t=\rho\sum_{i\in I}\sum_{n\in S_i}\left(\boldsymbol{e}_{n,i}^t-\boldsymbol{e}_{n,i}^{t-1}\right) \tag{23}$$

The above iteration is performed until the Euclidean norm of both the primal and dual residuals are smaller than thresholds.

In the second-stage benefit allocation planning problem, Eq. (12) becomes the coupling constraint.

## 4.3. Release and Fix method

To improve the convergence performance of the consensus ADMM in the first-stage MILP problem, the Release and Fix method [8], which is a heuristic to find a near-optimal solution of MILP problems through ADMM iterative calculation, is combined with the consensus ADMM. As is the case with Reference [7], the consensus ADMM is applied in two steps. The first-step and second-step consensus ADMMs are referred to as consensus ADMM-Release and consensus ADMM-Fix, respectively. In the first-step consensus ADMM-Release, the consensus ADMM is directly applied to the coordinated operational planning problem of Eq. (14) to determine the values of all the binary variables. Thus, each subproblem results in a mixed-integer quadratic programming problem. In the second step, the subproblems, in which the values of the binary variables are fixed at the result of the consensus ADMM-Release, are generated; consequently, each subproblems results in a quadratic programming (QP) problem. By solving QP-based problems with the consensus ADMM, a better solution is searched. The solution of the binary variables and the continuous variables, obtained in the consensus ADMM-Release and consensus ADMM-Fix, respectively, is regarded as a near-optimal solution of the coordinated operational planning problem. The Release and Fix method is applied to only the coordinated operational planning problem because the benefit allocation planning problem does not have integer variables.

# 5. Case study

## 5.1. Calculation condition

### 5.1.1. P2P energy trading network

The configuration of the target P2P energy trading network in the case study is the same as Fig. 1. The number of prosumers and consumer is four, i.e., $N$ = 4. All the prosumers and consumer install a heat pump water heater, of which the rated heat output is 4.5 kW, and a hot water storage tank, of which the volume is 460 L. In Prosumers #1 and #2, PV panel are installed. In Prosumers #2 and #3, battery units are installed. The capacity of the PV panels and battery units are listed in Table 1. The capacity is selected so that the P2P energy trading is actively performed. For the performance characteristics of the PV panels, the heat pump water heaters, hot water storage tanks, and battery units, refer to our previous publication [13].

Table 1 Specifications of PV panel and battery unit

| Eqiupment | Installed prosumer | *Value* |
|---|---|---|
| Photovoltaic panel area | Prosumer #1 | 20.0 $m^2$ |
| | Prosumer #2 | 15.0 $m^2$ |
| Battery unit | Prosumer #2 | 5.0 kWh |
| | Prosumer #3 | 15.0 kWh |

### 5.1.2. Input condition

As stated in Section 3.1, the PV output, the energy demand of the prosumers and consumer, and the time-variant purchasing and selling prices with a power retailer are given as the input condition. The planning horizon length is 24 hours, and its sampling time interval $\varDelta T$ is 1 hour; thus, $K$ = 24. The weighting parameters, $\varphi_{\mathrm{PV}}$, $\varphi_{\mathrm{P2P}}$, and $\varphi_{\mathrm{BT}}$, in Eq. (9) are set at 0.3, 0.4, 0.3, respectively.

The simulated result of the PV output using measured solar radiation data on a sunny weekday in mid-season [9] is used. The electric power and hot water demands of four dwellings were measured in a housing complex in Japan [9]. The purchasing price from a power retailer is time-variant. The price structure has three zones: a daytime zone (7:00–10:00 and 17:00–23:00) with 27.32 JPY/kWh, a peak-time zone (10:00–17:00) with 35.54 JPY/kWh, and a night-time zone (23:00–7:00) with 13.10 JPY/kWh. For its details, refer to our previous publication [13]. The selling price of the surplus PV output is set at 8 JPY/kWh; this assumes that the feed-in tariff period has expired.

### 5.1.3. Computational condition

The thresholds for the primal and dual residuals to terminate the iterative calculation of the consensus ADMM are set at 1.0×$10^{-3}$ kW. The upper limit of the iteration number is 200.

All the optimization problems were coded by using the algebraic modelling language GAMS distribution 49.1 and solved by using the commercial optimization solver Gurobi version 12.0.1.

## 5.2. Results and discussion

### 5.2.1 First-stage coordinated operational planning analysis

The result of the first-stage coordinated operational planning on a sunny weekday in mid-season is shown in Fig. 4. The PV output utilization in Prosumers #1 and #2, the battery charging and discharging power and stored electricity in Prosumers #2 and #3, and the exporting and importing power of all the prosumers and consumer are focused on. On this sunny day, a large amount of surplus PV output is generated because the electric power demand including the power consumption of the heat pump water heaters is small. The surplus PV outputs are used for the P2P energy trading as well as the battery charging and the electric power selling. Most of the surplus PV outputs imported from Prosumers #1 and #2 are charged in the large-capacity battery of Prosumer #3. This charged electricity is exported to other prosumers and consumer in the night-time. By conducting the battery storage, the surplus PV output generated in the daytime can be utilized effectively in the P2P energy trading network. The surplus PV output in Prosumer #2 is slightly sold to the power retailer because the electricity stored in the battery of Prosumers #2 and #3 reaches the upper limit during the daytime.

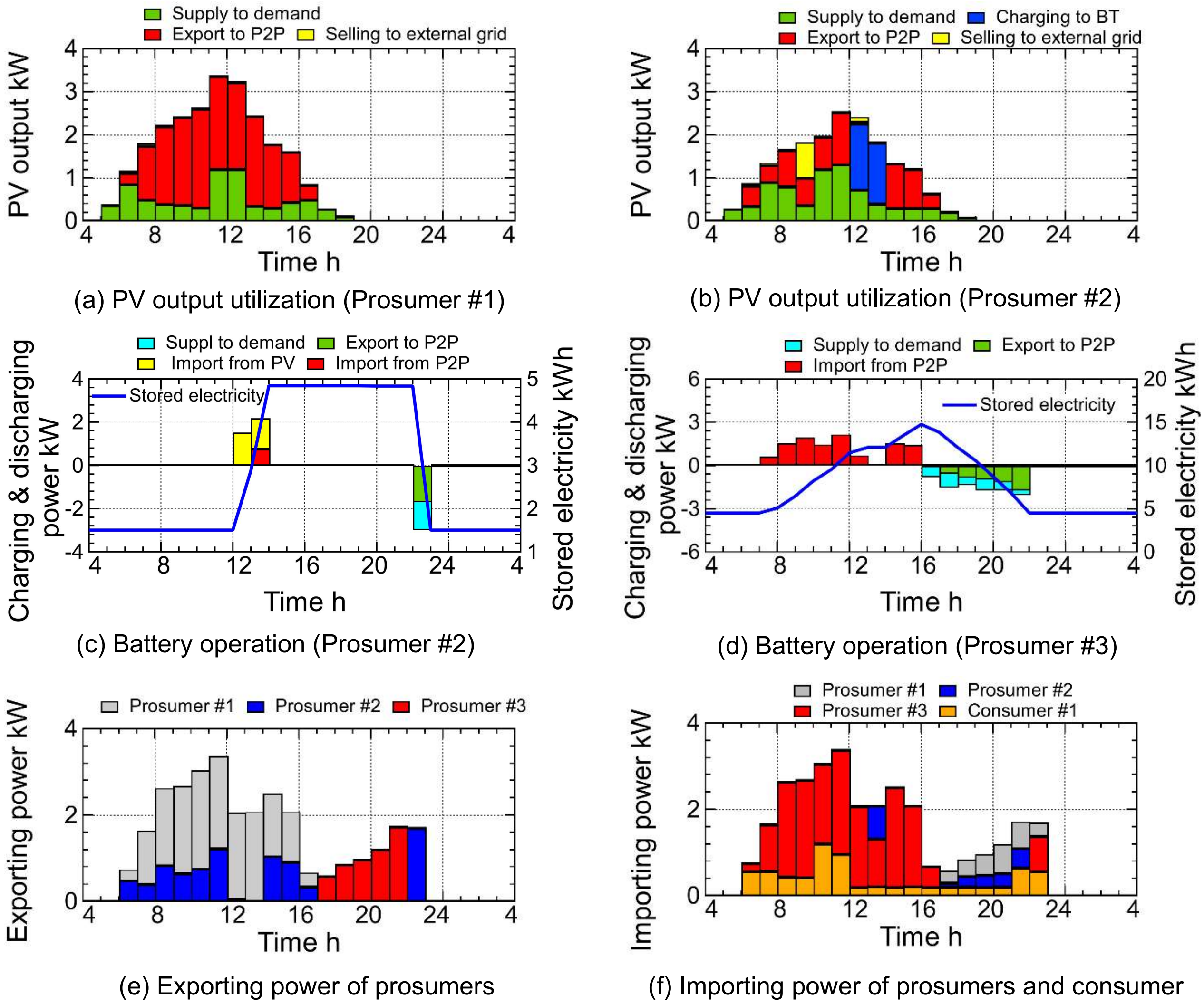



**Figure. 4.** First-stage coordinated operational planning result.

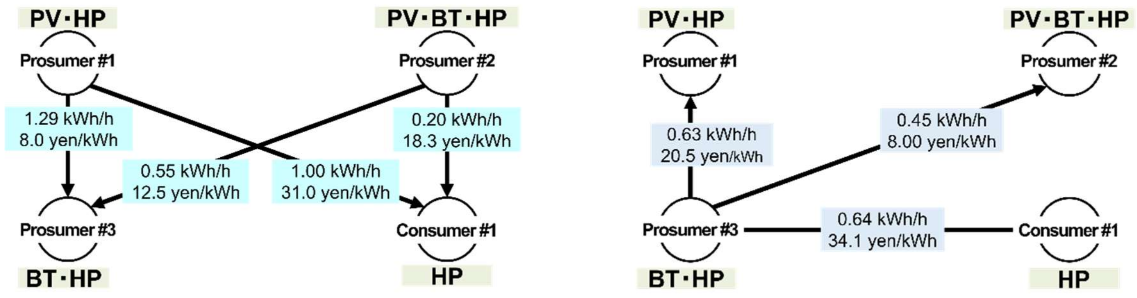
### 5.2.2 P2P energy trading contribution analysis

The P2P energy trading contributions and resulting bargaining power of the prosumers and consumer, which were calculated from the result of the first-stage coordinated operational planning, are listed in Table 2. Prosumers #1 and #2 show high values of the PV output utilization ratio $R_n^{\mathrm{PV}}$, and Prosumer #2 and #3 have high values of the battery utilization ratio $R_n^{\mathrm{BT}}$. The engagement ratio in the P2P energy trading $R_n^{P2P}$ is increased in Prosumers #1 and #3 leveraging the PV panel and the large-capacity battery. The bargaining power based on these contributions is strong in Prosumer #1, which greatly contributes to PV output utilization and P2P energy trading, and Prosumer #3, which utilizes the battery. The bargaining power of Prosumer #2, installing the PV panel and battery, is not so strong due to small engagement ratio in the P2P energy trading.

### 5.2.3 Second-stage benefit allocation planning analysis

Table 2 P2P energy trading contributions and bargaining power

| Prosumer | P2P energy trading contributions | | | Bargaining power |
|---|---|---|---|---|
| | PV output utilization $R_n^{\mathrm{PV}}$ | P2P trading engagement $R_n^{P2P}$ | Battery utilization $R_n^{\mathrm{BT}}$ | |
| Prosumer #1 | 0.697 | 0.323 | 0 | 0.310 |
| Prosumer #2 | 0.365 | 0.172 | 0.366 | 0.269 |
| Prosumer #3 | 0 | 0.389 | 0.810 | 0.372 |
| Consumer #1 | 0 | 0.116 | 0 | 0.043 |




(a) Daytime (10:00–11:00)  (b) Night-time (21:00–22:00)

**Figure. 5.** P2P energy trading result in second-stage benefit allocation planning.

Figure 5 shows examples of the P2P energy trading planning, i.e., trading power and price, at 10:00–11:00 (daytime) and 22:00–23:00 (night-time), which were obtained by solving the second-stage benefit allocation planning problem with the bargaining power in Table 2.

In the daytime, the surplus PV output of Prosumers #1 and #2 is exported to Prosumer #3 and Consumer #1. The trading price from Prosumer #1 to Prosumer #3 is low, i.e., 8 JPY/kWh, while that to Consumer #1 is high, i.e., 31.0 JPY/kWh. This is due to the relative relationship of the bargaining powers; the bargaining powers of Prosumer #3 and Consumer #1 are stronger and weaker than that of Prosumer #1, respectively. Although the power export to Consumer #1 is conducted with high trading prices, these prices are more economical than the purchased price from a power retailer, i.e., 35.54 JPY/kWh; this contributes to the reduction in the energy cost of Consumer #1. At night, the charged power of Prosumer #3 is exported to Prosumers #1 and #2 and Consumer #1. Unlike the P2P energy trading in the daytime, the correlation between the bargaining power and the P2P energy trading price is hardly observed. This is because the bargaining powers affect the daily total amount of trading energy.

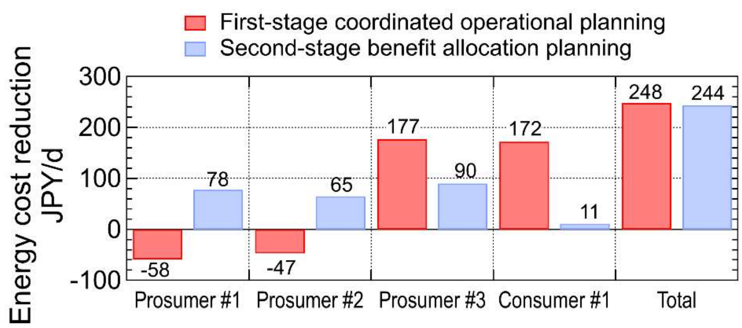



**Figure. 6.** Energy cost reduction in first-stage and second-stage planning.

The energy cost reduction from the individual operation in the first-stage coordinated operational planning and second-stage benefit allocation planning is shown in Fig. 6. In the first-stage coordinated operational planning, the energy cost reduction of Prosumers #1 and #2 installing the PV panels shows negative values, while that of Prosumer #3 and Consumer #1 is very high. This is because the first-stage coordinated operational planning minimizes the total energy cost of the P2P energy trading network without considering the P2P energy trading charge. By performing the second-stage benefit allocation planning, the energy cost reduction presents a similar trend of the bargaining power listed in Table 2. The slight difference in the total energy cost reduction between the first and second stages is caused by adjusting the primal residual expressing the satisfaction of the coupling constraint of Eq. (16).

### 5.2.4 Comparison between centralized optimization and distributed optimization

The energy cost reduction from the individual operation, which was obtained by performing the centralized optimization (Fig. 2) and the distributed optimization (Fig. 3), is shown in Fig. 7. It was confirmed that the centralized optimization result is the optimal solution. Although the energy cost reduction of each prosumer and consumer is slightly different between the centralized optimization and the distributed optimization, the difference in the total energy cost reduction is 2 JPY/d, which is much smaller than the total energy cost of the

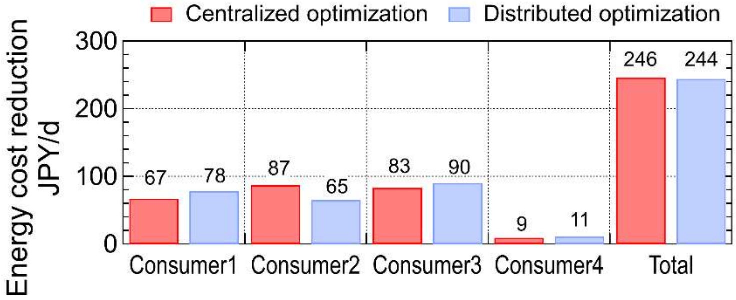



**Figure. 7.** Comparison in energy cost reduction between centralized optimization and distributed optimization.

individual operation, i.e., 394.4 JPY/d. Thus, the P2P energy trading planning based on the two-stage distributed optimization is regarded to be a near-optimal solution.

# 6. Conclusions

A two-stage distributed optimization method to efficiently determine the energy trading amount and the energy trading price was developed for P2P energy trading networks. The developed method is based on a two-stage optimization, in which the original problem is decomposed into a first-stage MILP-based coordinated operational planning problem without considering the P2P energy trading charge and a second-stage NLP-based benefit allocation planning problem using asymmetric Nash bargaining and the bargaining power calculated by the contribution to the P2P energy trading. To perform the two-stage optimization in a distributed manner from the perspective of information security and problem scalability, the consensus ADMM for multiple subproblems and the Release and Fix method for non-convex subproblems with integer variables are uniquely combined.

The developed method was applied to P2P energy trading in a residential network composed of three prosumers and one consumer. The results showed that the energy trading amount and price are properly determined under the given PV output variations and energy demands and the calculated P2P energy trading contribution. Moreover, the distributively optimal solution was close to the centrally optimal solution. This means that high energy cost reduction and fair benefit allocation can be achieved by using the developed method. Although the results could not be presented due to the page limitation, it was confirmed that the consensus ADMM is well converged, and the Release & Fix method can find a near-optimal mixed-integer solution satisfying the P2P energy trading constraints.

The developed method has several issues. First, the developed method should be applied to various operating conditions, i.e., PV outputs and energy demands. Second, the scalability of the optimization problems should be demonstrated by applying it to P2P energy trading networks with multiple prosumers and consumers.

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